Engineering Notes

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Experimental-Theoretical Correlation of Supersonic Jet-on Base Pressure for Cylindrical Afterbodies

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Nomenclature

C = Crocco number
M = Mach number
P = absolute pressure
r = empirical recompression coefficient
$I\!R$ = gas constant
T = absolute temperature
X, R = longitudinal and radial coordinate, respectively
β = flow angle
γ = ratio of specific heats
$\bar{B}_0$ = base bleed-to-nozzle mass-flow rate ratio
$\bar{P}_{1I}$ = nozzle exit plane-to-freestream static pressure ratio, P_{1I}/P_{1E}
$\overline{I\!R}_I$ = ratio of gas constants, $I\!R_I/I\!R_E$
$\bar{T}_{0E}$ = external-to-internal stream stagnation temperature ratio, T_{0E}/T_{0I}
$\bar{X}_{1I}$, $\bar{R}_{1I}$ = dimensionless coordinate of the separation point of the internal stream: X_{1I}/R_{1E}, R_{1I}/R_{1E}
$\bar{\delta}^*$, $\bar{\delta}^{**}$ = boundary-layer displacement and momentum thickness-to-radius ratio, δ^*/R_{1E}, δ^{**}/R_{1E}

Subscripts

B = base region
d = discriminating stream line
E = external (freestream) flow
I = internal (nozzle) flow
S = oblique-shock recompression system
0 = stagnation condition
1 = separation point

Introduction

BASE drag constitutes a significant part of the over-all vehicle drag under powered supersonic flight conditions; as a consequence, proper evaluation of the base drag is essential for meaningful aerodynamic evaluation, design, or optimization studies. The component flow model[1,2] of Korst et al., is well suited for these studies since it offers a tenable method for predicting the influence and relative significance of the many flow and geometrical variables involved. In addition, the component aspects of this flow model are readily adaptable to computerized analyses.[2]

Experience has shown that general trends and the functional dependence of base-pressure and base-temperature ratios on the governing variables can be predicted correctly with this flow model.[2,3] There are, however, geometrical and flow conditions wherein significant discrepancies exist between the level of predicted and experimental values of the base-pressure ratio; these discrepancies are most pronounced for those cases in which the nozzle is relatively small in comparison with the base.[2]

In the component flow model, the recompression process is central in determining the base-flow solution since it links the "corresponding" inviscid flowfield and turbulent mixing components. Since the recompression process is not well understood and strongly influences the solution to the base-flow problem, the need to improve agreement between predicted and experimental values has resulted in several empirical modifications to this part of the flow model.

For a single two-dimensional supersonic stream reattaching at a solid boundary, Nash[4] proposed and determined an empirical "reattachment condition" which was approximately independent of the supersonic approach Mach number. Carrière and Sirieix[5] proposed and determined an empirical law of reattachment, for a negligible initial boundary layer, based on "angular criterion of turbulent reattachment." Their law of reattachment correlates well the data for reattachment at a wall of supersonic axisymmetric, two-dimensional, and conical flows. Page, Kessler, and Hill[6] offer an alternative empirical reattachment criterion for two-dimensional supersonic flows based on the "discriminating-streamline" velocity ratio.

For the axisymmetric two-stream base-flow problem, the recompression resulting from the interaction of the separated supersonic freestream-nozzle flows apparently cannot be determined by a direct carry over of the empirical reattachment criteria determined in the aforementioned investigations. Sirieix, Délery, and Mirande[7] have presented a more generalized "criterion of turbulent reattachment" involving geometric and flow variables. More recently, Dixon, Richardson, and Page[8] assumed that the external inviscid flow boundary remains straight after initially turning to adjust to the base pressure. This assumption results in an axial-pressure gradient from which the internal flow boundary is determined. The "corresponding" inviscid flowfields are linked to an approximate axisymmetric mixing component by a recompression criterion based on Goethert's modification of Korst's "escape criterion." Both of these modifications improved the agreement between predicted and experimental values of base pressure for the limited data presented.[7,8]

With this background and the availability of a computer program[2] based on the flow model of Korst et al., an alterna-

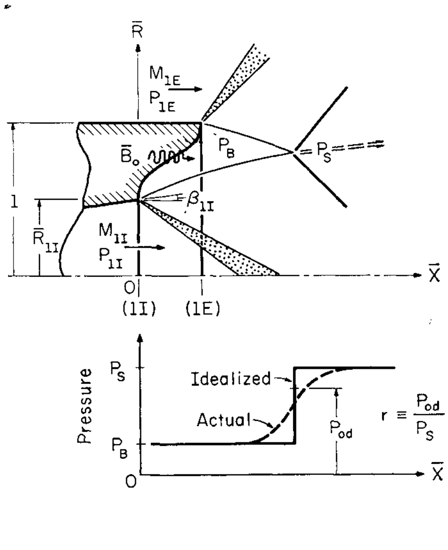

Fig. 1 Two-stream axisymmetric base-flow configuration, notation, and recompression-coefficient definition.

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Table 1 Summary of experimental data on which the recompression coefficient correlation is based

	References	Data[a]	M_{1E}	M_{1I}	β_{1I}	$\bar{R}_{1I}$
9	Cortright and Schroeder	...	1.91	1.0	0°	0.5
10	Messing, Rabb, and Disher	$\bar{T}_{0E} = 0.141$, $\gamma_I = 1.22$	1.5–5.15	2.93	15°	0.555
11	Bromm and O'Donnell	...	1.62, 1.94, 2.41, 3.12	1.0, 3.0	0°, 10°, 20°	0.5, 0.75
12	Baughman and Kochendorfer	...	1.91, 3.12	1.0	0°	0.375
13	Reid and Hastings	...	2.0	2.0	0°, 5°, 10°	0.2, 0.4, 0.6, 0.8
14	Henderson	...	1.65, 1.82, 2.21	2.5	0°, 15°	0.333, 0.455
15	Craven et al.	...	2.0	1.0, 1.4, 2.0	0°	0.188
16	Charczenko and Hayes	$\bar{X}_{1I} = 1.0$	2.3, 2.95, 4.0, 4.65	3.18	10°	0.437
17	Roberts and Golesworthy	$0 \leq \bar{B}_0 \leq 0.06$	2.20	2.23	10°	0.772
18	White	$0 \leq \bar{B}_0 \leq 0.08$, $P_{0I}/P_{1E} = 30, 112$	2.5	2.7	20°	0.20
19	Harries	...	1.5, 2.01	1.0, 1.5	0°, 10°	0.200, 0.333
20	Reid	$0.333 \leq \bar{T}_{0E} \leq 1.0$, $1.33 \leq \gamma_I \leq 1.4$, $IR_I = 97$ lb$_f$-ft/lb$_m$-°R	2.0	1.0, 2.0	0°	0.5, 0.645
21	Brazzel	...	1.5, 2.5, 2.87, 3.0	1.0, 1.78, 2.7, 3.8	0°, 10°, 20°	0.1, 0.141, 0.2, 0.3, 0.45, 0.8
22	Univ. of Illinois, U-C	$\gamma = 1.22, 1.4, 1.67$	2.0, 2.5	1.0, 1.78, 1.88, 2.2, 2.41, 2.44, 2.7, 2.8, 3.09, 3.2, 3.8, 3.89	0°, 5°, 10°, 15°, 20°	0.100, 0.125, 0.141, 0.168, 0.175, 0.200, 0.210, 0.250, 0.252, 0.300, 0.314, 0.320, 0.375, 0.400

[a] Unless otherwise noted, the following variables are the same for all configurations and have the values: $IR_E = IR_I = 53.35$(lb$_f$-ft/lb$_m$-°R), $\bar{X}_{1E} = \bar{X}_{1I} = 0.0$, $\bar{T}_{0E} = 1.0$, $\gamma_E = \gamma_I = 1.4$, $\bar{R}_{1E} = 1.0$, $\bar{B}_0 = 0.0$.

tive and somewhat simpler empirical flow-model modification than the foregoing is proposed herein. The recompression criterion is modified by assuming that the discriminating streamlines or the jet-boundary streamlines, as the case may be, are capable of recompressing by stagnating only to a fraction of the pressure rise from the base region to the downstream shock region as determined for the "corresponding" inviscid flowfields. The unknown fractional pressure rise is then determined by a detailed correlation of experimental and theoretical data.

Empirical Flow-Model Modification

The configuration and the associated notation is shown in Fig. 1. The internal and external streams separate at locations (1I) and (1E), respectively, and subsequently interact forming a shock recompression system at the impingement point of the streams. The recompression pressure rise is the basis for distinguishing between the fluid energized by the mixing which has sufficient mechanical energy to recompress to this pressure level and that which has insufficient mechanical energy and, as a consequence, must recirculate into the base region. The base-flow solution is then determined by the requirement of conservation of mass and energy for the over-all base region.

If the pressure rise immediately downstream of the oblique shock system (Fig. 1) is given relative to the base pressure as (P_S/P_B), then the pressure level to which the discriminating streamline must recompress by stagnating is assumed to be given by

$$P_{0d}/P_d = (1 - C_d^2)^{-[\gamma/(\gamma-1)]} = r(P_S/P_B) > 1 \quad (1)$$

Equation (1) defines the empirical recompression coefficient r.

At the outset, it is reasonable to assume that r could be a function of the numerous dimensionless groups governing this problem. In general, the functional form of r could be expressed as

$$r = f(\bar{X}_{1I}, \bar{R}_{1I}, \beta_{1I}, M_{1I}, \gamma_I, \overline{IR}_I, M_{1E}, \gamma_E, \bar{T}_{0E}, \bar{B}_0, \bar{P}_{1I}, \bar{\delta}_{1I}^*, \bar{\delta}_{1I}^{**}, \bar{\delta}_{1E}^*, \bar{\delta}_{1E}^{**}) \quad (2)$$

If r were indeed strongly dependent on all the foregoing dimensionless variables, the possibility of determining its functional form, of course, would be remote. However, since the flow model considers and apparently adequately accounts for many of the variables in the inviscid, viscid, or recompression components of the flow model, the influence on r of some of the variables would be expected to be small.

Determination of the form of Eq. (2) then must be based on a detailed comparison between predicted and experimental data. Most of the experimental data reported in the literature and determined in connection with the continuing study of the base-flow problem[9–22] have been obtained in small scale, cold-flow, air-to-air model studies. Generally, the experimental programs and available data can be classed according to the grouping and variation in the variables of Eq. (2).

Much experimental data have been obtained in studies wherein variations were made principally in $[\bar{R}_{1I}, \beta_{1I}, M_{1I}, M_{1E}, \bar{P}_{1I}, \bar{B}_0]$. Data for the more difficult experimental studies dealing with variations in $[\gamma_I, \overline{IR}_I, \bar{T}_{0E}]$ are limited in quantity and scope. An assessment of the influence of

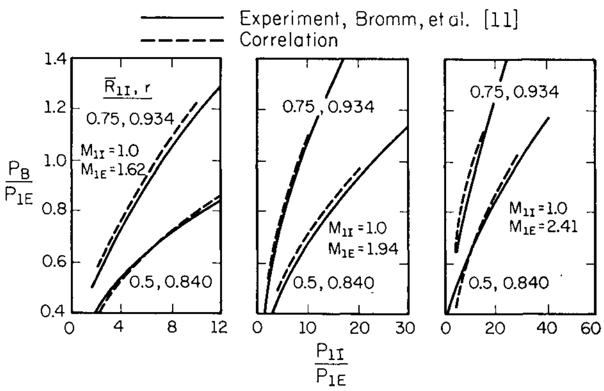

Fig. 2 Typical experimental-theoretical correlation with the recompression coefficient.

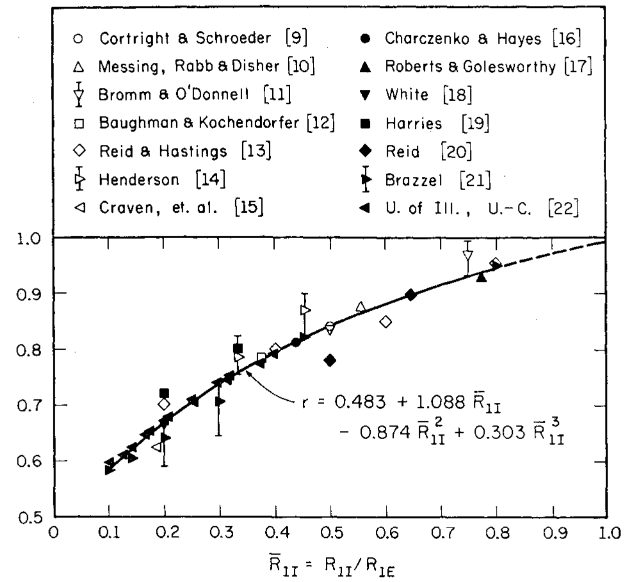

Fig. 3 Empirical recompression coefficient.

boundary layers, which are characterized by [$\bar{\delta}_{1I}^*, \bar{\delta}_{1I}^{**}, \bar{\delta}_{1E}^*, \bar{\delta}_{1E}^{**}$] is complicated by the lack of complete data on the boundary layers or by the small scale of models for which boundary-layer effects are small or have been reduced intentionally. As a consequence, only a qualitative indication of the causal relationship between the base-pressure ratio and the boundary layers can be inferred.

The sources and the variation(s) in the variable(s) for experimental data used in the determination of the recompression coefficient are summarized in Table 1. Values of r were determined, for a given configuration, by systematically varying r until a value of r was determined which would give reasonably "good" agreement between predicted and experimental data throughout the range of the principal independent variable which, in most cases, was the nozzle-to-freestream pressure ratio. Figure 2 illustrates typically the degree of correlation that can be achieved between experimental and theoretical data for a given configuration with a single value of the recompression coefficient.

Detailed comparisons[3] with approximately 150 sets of experimental data have shown that the recompression coefficient r is principally a function of the nozzle-to-base radius ratio. Values of the recompression coefficient thus determined for the experiments summarized in Table 1 are presented in Fig. 3 as a function of the nozzle-to-base radius ratio $\bar{R}_{1I}$. In cases wherein adequate experimental data were available, it was possible to determine the maximum variation in r for a given value of $\bar{R}_{1I}$; these variations are indicated in Fig. 3. Some variation in values of r can be attributed to the dependence, although small, of the recompression coefficient on the other variables, as well as the boundary layers which have not been explicitly considered in this study.

Based on values of the recompression coefficient presented in Fig. 3, a convenient least-squares representation of the recompression coefficient is given by

$$r = 0.483 + 1.088\bar{R}_{1I} - 0.874\bar{R}_{1I}^2 + 0.303\bar{R}_{1I}^3 \qquad (3)$$

The standard error for this curve fit is $s_r = 0.0151$. Equation (3) is also plotted in Fig. 3.

Conclusions

For purposes of engineering analyses, the modification of the flow model of Korst et al., by an empirically determined recompression coefficient is well justified as a result of the excellent correlation achieved between experiment and theory over a wide range of experimental variables. As a consequence, the quantitative evaluation of the base drag utilizing the recompression coefficient presented herein can be made with a degree of confidence. In addition, the recompression-coefficient correlation also provides a basis for a unified comparison between the diversity of experimental base-flow data.

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Wave Drag of Optimum and Other Boat Tails

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1. Introduction

MINIMIZING the boat tail drag of an aircraft or a missile is a problem of considerable interest to the design engineer. Two particular aspects of the problem are considered in this Note. First, the shapes of minimum-drag boat tails were determined for supersonic flow. Secondly, the wave drag coefficients of these optimum bodies were compared to the drag coefficients of other commonly used boat tail contours such as conical, circular arc and parabolic arc.

2. Minimum-Drag Boat Tails

The problem of optimizing the boat tail shape can be posed as illustrated in Fig. 1. Given the length and radii of the boat tail and the ambient Mach number, what contour connecting points A and B will result in the least wave drag?

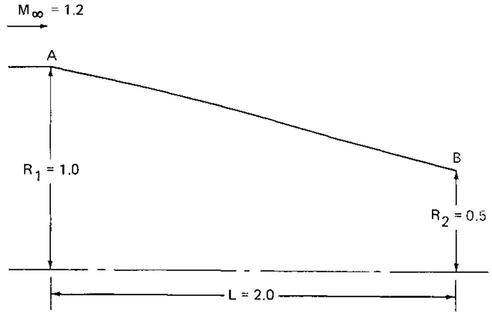

Fig. 1 Optimum boat tail contour for $L/R_1 = 2.0$, $R_2/R_1 = 0.5$, and $M_\infty = 1.2$.

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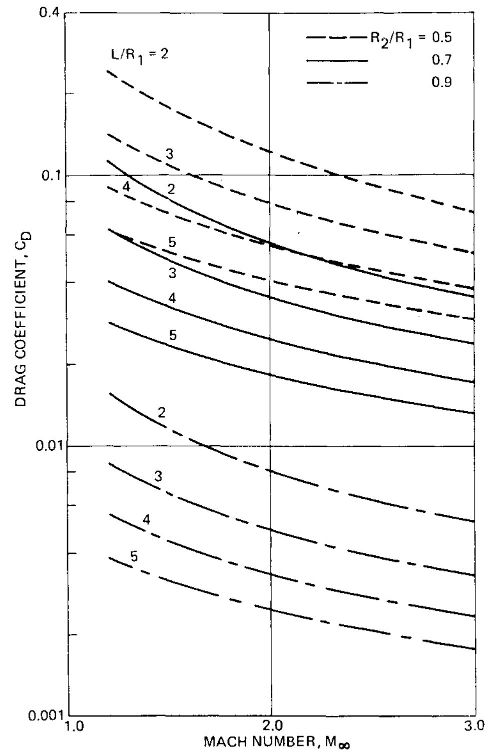

Fig. 2 Drag coefficients of optimum boat tails.

The appropriate way to analyze problems of this type is by way of calculus of variations. In searching the literature, it was discovered that although the problem had not been treated exactly, it had been solved for linearized supersonic flow. (No attempt at optimization using the full nonlinear equations was made within the scope of the present effort). The linearized analysis of Parker[1] considered the problem of minimizing the transition section drag connecting two axisymmetric cylinders with the larger one located downstream. Since linearized equations were used, the flow direction can be reversed and the solution would remain unchanged to the first order. Thus, the results of Parker's analysis are directly applicable to the boat tail problem. Parker does not arrive at an explicit relationship $R = f(x)$, but rather derives an integral equation relating R and x

$$\beta^2(R^2 - R_2^2) = \frac{4\beta^2(R_1^2 - R_2^2)(1 + \beta R_1 + \beta R_2)}{\pi(1 - \beta R_1 + \beta R_2\ (1 + 3\beta R_1 + \beta R_2)} \times \int_{\pi}^{\cos^{-1}\left[\frac{2(x - \beta R) - (1 + \beta R_1 - \beta R_2)}{1 + \beta R_1 + \beta R_2}\right]} \cos\theta \left\{\left[x - \frac{1 + \beta R_1 - \beta R_2}{2} - \frac{(1 + \beta R_1 + \beta R_2)}{2}\cos\theta\right]^2 - \beta^2 R^2\right\}^{1/2} d\theta \quad (1)$$

where

$$\beta = (M_\infty^2 - 1)^{1/2} \quad (2)$$

A computer program was written to solve this equation numerically. Thus, for given values of R_1, R_2, L, and M_∞ the program could generate a table of R values corresponding to a selected range of x values.

The previous program was used to generate minimum drag profiles for boat tails having thickness and length ratios of practical interest. Thickness ratios (R_2/R_1) of 0.5, 0.7, and 0.9 and length ratios (L/R_1) of 2, 3, 4, and 5 were selected for these computations. The freestream Mach number was taken equal to 1.2, 2.0, and 3.0. It was observed that, although the computed contours were curved and exhibited an inflection point, they were all rather close to conical in